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SPECTRAL DISPERSION OF CLOUD DROPLET SIZE DISTRIBUTIONS AND THE PARAMETERIZATION OF CLOUD DROPLET EFFECTIVE RADIUS

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Spectral dispersion of cloud droplet size distributions and the parameterization of cloud droplet effective radius

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Abstract. Parameterization of effective radius (r_e) as proportional to the cube root of the ratio of cloud liquid water content (L) to droplet concentration (N), i.e., $r_e = \alpha(L/N)^{1/3}$, is becoming widely accepted. The principal distinction between different parameterization schemes lies in the specification of the prefactor α. This work focuses on the dependence of α on the spectral dispersion of the cloud droplet size distribution. Relationships by *Pontikis and Hicks* [1992] and by *Liu and Hallet* [1997] that account for the dependence of α on the spectral dispersion are compared to each other and to cloud microphysical data collected during two recent field studies. The expression of Liu and Hallet describes the spectral dependence of α (or r_e) more accurately than the Pontikis and Hicks relation over the observed range of spectral dispersions. The comparison shows that the different treatments of α as a function of spectral dispersion alone can result in substantial differences in r_e estimated from different parameterization schemes, suggesting that accurately representing r_e in climate models requires predicting α in addition to L and N.

1. Introduction

Cloud droplet effective radius r_e (defined as the ratio of the third to the second moment of a droplet size distribution) is one of the key variables that are used for the calculation of the radiative properties of liquid water clouds [*Hansen and Travis*, 1974; *Slingo*, 1989]. The inclusion and treatment of r_e in climate models has proven to be critical for assessing global climate change. *Slingo* [1990] studied the sensitivity of the global radiation budget to r_e and found that the warming effect of doubling the CO_2 concentration could be offset by reducing r_e by approximately 2 µm. *Kiehl* [1994] found that a number of known biases of the early version of CCM2 were diminished, and important changes in cloud radiative forcing, precipitation, and surface temperature resulted, if different values of r_e were assigned to warm maritime and continental clouds. A high sensitivity to the method of parameterizing r_e was also found in a recent study of the French Community Climate model [*Dandin et al.*, 1997].

Early parameterization schemes expressed r_e as either a linear or a cube root function of the liquid water content, implicitly assuming no dependence of r_e upon total droplet concentration [*Stephens*, 1978; *Fouquart et al.*, 1989]. There has been increasing support for parameterizing r_e as a "1/3" power law of the ratio of the cloud liquid water content to the droplet concentration [*Pontikis and Hicks*, 1992; *Bower and Choularton*, 1992; *Bower et al.*, 1994; *Martin et al.*, 1994; *Liu and Hallett*, 1997; *Reid et al.*, 1998]. The "1/3" power-law takes the form

$$r_e = \alpha \left(\frac{L}{N} \right)^{1/3} \tag{1}$$

where r_e is the effective radius in µm, L the liquid water content in gm^{-3}, N the total droplet concentration in cm^{-3}, and α the prefactor.

A key issue in use of this parameterization is the specification of α. Here we explore the dependence of α on the spectral dispersion of cloud droplet size distributions. Values of α derived from other studies are compared to those derived from the data collected during two Intensive Observation Periods (IOP) conducted at the Atmospheric Radiation Measurements (ARM) program Southern Great Plain (SGP) site in Oklahoma, in the spring and fall of 1997. This analysis suggests the necessity and possibility of improving the representation of clouds in climate models by specifying α in addition to L and N.

2. Expressions For α

For clouds with a monodisperse droplet size distribution as described by a delta function $n(r)=N\delta(r-r_e)$, $\alpha = 100(3/4\pi)^{1/3} \approx 62.04$; the multiplier 100 is introduced to keep the units of r_e, L and N in µm, g m^{-3} and cm^{-3}, respectively. This value of α was used by *Bower and Choularton* [1992], and *Bower et al.* [1994] to estimate the r_e of layer clouds and small cumuli, where entrainment and mixing processes are minimal. In a study of the sensitivity of NCAR's CCM2 to variations in r_e, *Kiehl* [1994] used this scheme to provide support for choosing r_e of 5 µm and 10 µm for continental and maritime clouds respectively. However, monodisperse droplet size distributions seldom occur in real clouds; broader size distributions were reported even for clouds that are nearly adiabatic [*Brenguier and Chaumat*, 1999]. *Martin et al.* [1994] derived estimates of α of 66.83 for maritime, and 70.89 for continental stratocumulus clouds with small entrainment and mixing. Martin et al.'s scheme has been recently used to specify cloud properties in climate models [*Ghan et al.*, 1997; *Lohmann et al.*, 1999], and to address the indirect effect of aerosols on climate [*Rotstayn*, 1999]. However, Martin et al.'s scheme includes no explicit dependence of α on the spectral broadening processes that we believe to be important for specifying cloud properties in climate models.

Although it has been realized that α depends on spectral broadening processes such as entrainment and mixing, research on this dependency is very limited. By assuming a negligible skewness of the droplet size distributions, *Pontikis and Hicks* [1992] analytically derived an expression (P-H and α_{PH} hereafter) that relates α to the spectral dispersion *d*, viz,

$$\alpha_{PH}(d) = 62.04 \frac{\left(1+3d^2\right)^{2/3}}{\left(1+d^2\right)}, \tag{2}$$

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where d is defined as the ratio of the standard deviation to the mean radius of the corresponding droplet size distribution. Compared to using fixed values for α, the P-H expression improves the parameterization of r_e. However, as will be shown below, this parameterization is appropriate only for clouds with relatively narrow droplet size distributions. For clouds exhibiting broad size distributions, r_e is still underestimated. *Liu and Hallett* [1997] developed another "1/3" power-law expression from consideration of systems theory as applied to cloud droplet size distributions. This theory, whose details were described in *Liu et al*. [1995] and *Liu and Hallett* [1997; 1998], is built upon the principle of Shannon's maximum entropy. The resultant expression for α (L-H and α_{LH} hereafter) is given by

$$\alpha_{LH}(\mathrm{b}) = 64.52\frac{\Gamma^{2/3}(3/\mathrm{b})}{\Gamma(2/\mathrm{b})}\,\mathrm{b}^{1/3} \quad , \tag{3}$$

where $\Gamma(t) = \int_0^\infty z^{t-1}\exp(-z)dz$, and b is a parameter that depends on physical processes such as entrainment and mixing. It was demonstrated in *Liu and Hallett* [1997] that d decreases with increasing b. Although both the P-H and L-H expressions were proposed to quantify the effect of spectral broadening processes on the parameterization of r_e, they have not been compared. This is done in the next section.

3. Comparison of α_{PH} and α_{LH}

Figure 1 shows α_{PH} and α_{LH} as a function of the spectral dispersion d. Also shown in this figure are the α's for a monodisperse size distribution (MO), and Martin et al.'s values for maritime (MM) and continental (MC) clouds. The value of α_{PH} was calculated using Eq. (2). The value of α_{LH} was calculated using the relationship between b and d from the Weibull droplet size distribution derived from systems theory [*Liu and Hallett*, 1997],

$$d = \left[\frac{2\mathrm{b}\,\Gamma\left(2/\mathrm{b}\right)}{\Gamma^2\left(1/\mathrm{b}\right)} - 1\right]^{1/2} \quad . \tag{4}$$

For a given value of b, d was calculated using Eq. (4); this value of d was then substituted into Eq. (3) to obtain α. The relationship between α and d was determined by repeating this procedure for different values of b.

Significant differences between the dependencies of α_{PH} and α_{LH} on the dispersion are exhibited in Figure 1. The prefactor α_{LH} monotonically increases with the spectral dispersion, whereas α_{PH} reaches a maximum at a spectral dispersion of 1.0 and then starts to decrease. It is noteworthy that the α_{LH} and α_{PH} approach each other, and both approach the α for a monodisperse droplet size distribution, when the spectral dispersion approaches zero. The question arises as to the accuracy of these expressions. This question is addressed by analyzing cloud droplet size distributions measured during two recent IOPs at the ARM SGP site in northern Oklahoma in the spring and fall of 1997, respectively.

Spectral dispersions and α's were calculated from droplet size distributions collected at a rate of 1 Hz using a Forward Scattering Aerosol Spectrometer Probe (FSSP, Particle Measurement Systems Inc., Boulder, CO) mounted on the DOE Gulfstream-1 aircraft. Data from six flights in (broken) stratocumulus were analyzed, and are displayed in Figure 2. This figure shows that the values of α derived from the measurements increase monotonically with the spectral dispersion, and that this dependency more closely follows the dispersion dependency of α_{LH} rather than α_{PH}. Also note that many α's derived from the measurements are much larger than Martin et al.'s values of α, as well as the α for a monodisperse size distribution. This may be because spectral broadening processes in the clouds sampled during the IOP were much stronger than the spectral broadening associated with the clouds that Martin et al. considered in their analysis.

4. Comparison of Measured and Parameterized r_e

This section further illustrates the superiority of the L-H scheme by comparing values of r_e measured by the FSSP (r_{em}) with those estimated from the different parameterization schemes. To emphasize the effect of spectral dispersion and reduce the scatter, droplet size distributions from all the six flights were averaged according to their spectral dispersions (Data were first partitioned into groups within which droplet size distributions had similar spectral dispersions, and the data in each group were then averaged.) As indicated in Figure 3, the L-H scheme obviously outperforms the other schemes, which all underestimate r_e albeit to different degrees.

The substantial differences in parameterized values of r_e are due to the different treatments of α as a function of the spectral dispersion because the same values of L and N are used for all the parameterization schemes. This result can be better understood by examining the differences between r_{em} and parameterized r_e as a function of spectral dispersion. Figure 4 shows that except for the L-H scheme, the underestimation of r_e strongly increases with the spectral dispersion. At large spectral dispersions, the P-H scheme could underestimate values of r_e by as much as 3 μm; the underestimation is even larger for those schemes with fixed prefactors. To echo the introduction, such differences in r_e are large enough to cause noticeable errors in climate models. On the other hand, the underestimation of the L-H scheme is always within 1 μm and without obvious trend of change with the spectral dispersion.

It should be noted that the FSSP has both sizing and counting deficiencies [*Dye and Baumgardner*, 1984; *Baumgardner et al.*, 1985; *Baumgardner and Spowart*, 1990] which in turn can cause errors in the measurements of r_e [*Gerber*, 1996; *Wendisch*, 1998]. However, the focus of this study is on α, its dependence on the spectral dispersion, and its effect on the parameterized r_e given L and N. Any error in L and/or N will exert the same effect on all the parameterization schemes. Furthermore, the FSSP instrumental deficiencies are expected to have minimal effects on the relationship between prefactors and spectral dispersions derived from the FSSP-measured size distributions, because both are ratios of two quantities which are similarly affected ($\alpha = r_e/(L/N)^{1/3}$; d = mean radius/standard deviation). Therefore, the primary conclusions drawn from this study should hold regardless the FSSP instrumental deficiencies.

5. Conclusions

Existing "1/3" power-law expressions for parameterizing r_e in terms of ratio of liquid water content to droplet concentration are compared using data collected during two recent IOPs over the ARM SGP site. It is found that the Liu and Hallett scheme more accurately represents the dependence of prefactor on the cloud droplet spectral dispersion than the Ponkitis and Hicks scheme. The Liu and Hallett scheme appears to accurately represent the prefactor for clouds exhibiting a broad range of cloud droplet spectral dispersions, whereas the Ponkitis and Hicks scheme works well only when spectral dispersions are small, tending to underestimate the effective radius for clouds exhibiting broad size distributions. It is demonstrated that the Ponkitis and Hicks parameterization, along with the parameterization schemes with fixed prefactors, underestimate the effective radius, and that this bias could be large enough to cause serious problems in climate models.

Improvements in the representation of clouds in climate models have focussed on predicting liquid water content and droplet concentration. The effective droplet radius is then determined from the predicted liquid water content and droplet concentration by use of a "1/3" power-law with a fixed value of prefactor such as Martin et al.'s expression [*Smith*, 1990; *Ghan et al*., 1997; *Lohmann et al.*, 1999]. The effect of prefactor (spectral dispersion) on the effective radius has been barely documented. This study suggests that the prefactor is also important, and could be a decisive factor when droplet size distributions are very broad. Accurately representing r_e in climate models requires predicting the prefactor in addition to liquid water content and droplet concentration. The L-H expression or its equivalent may be considered as the first step to this goal.

A further step to predicting the prefactor may be taken by parameterizing the spectral dispersion, which is expected to depend on spectral broadening processes closely associated with turbulence intensity such as entrainment and mixing [*Cooper*, 1989]. This seems feasible by combining microphysical measurements of clouds with simultaneous measurements of turbulence such as cloud radar measurements [*Babb and Verlinde*, 1999].

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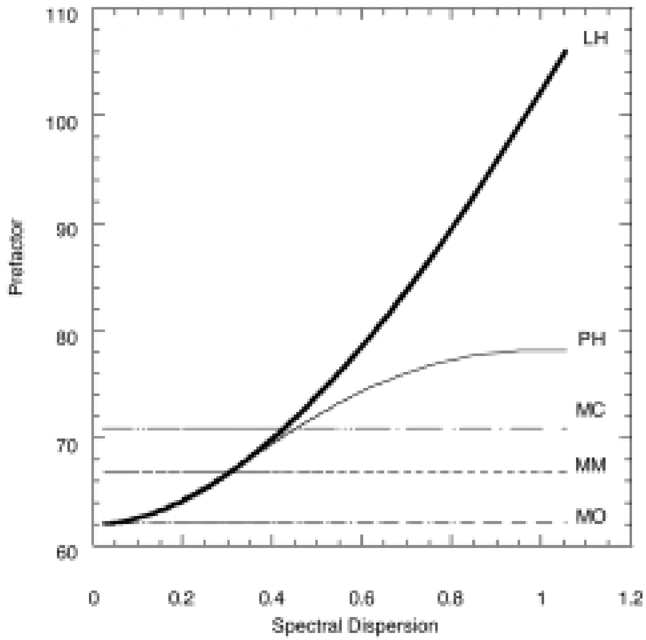

Figure 1. Dependency of the prefactor α on the spectral dispersion of the cloud droplet size distribution. LH and PH refer respectively to the Liu and Hallet and the Pontikis and Hicks expressions. MC, MM and MO refer to Martin et al.'s values of α for continental and marine clouds, and the value of α for monodisperse size distributions, respectively.

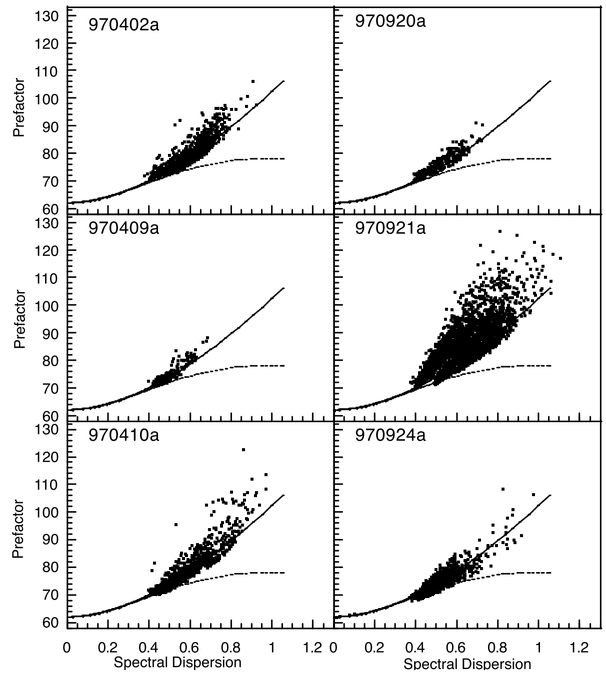

Figure 2. Comparison of prefactors calculated from the Liu and Hallett (solid curve) and Ponkitis and Hicks (dashed curve) expressions as a function of the spectral dispersion. The solid dots represent those derived from the FSSP-measured cloud droplet size distributions. The number on each plot such as "970420a" denotes flight numbers.

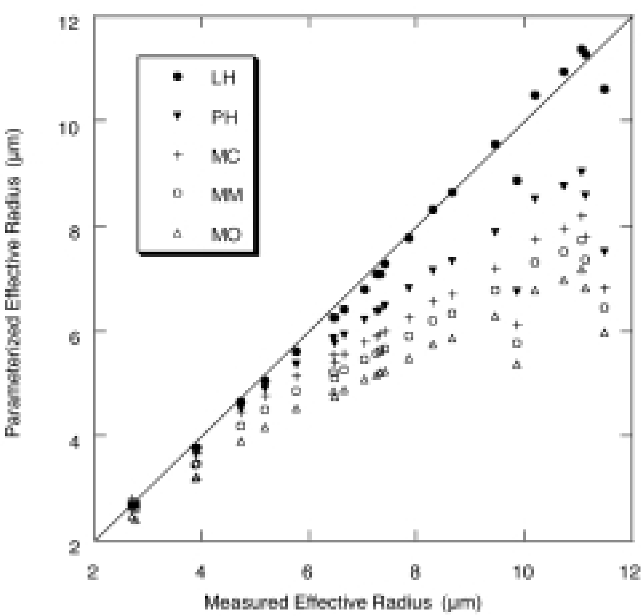

Figure 3. The cloud droplet effective radius estimated from the five different parameterization schemes as a function of the measured effective radius. LH, PH, MC, MM, and MO represent the effective radius estimated from the corresponding parameterization schemes, respectively.

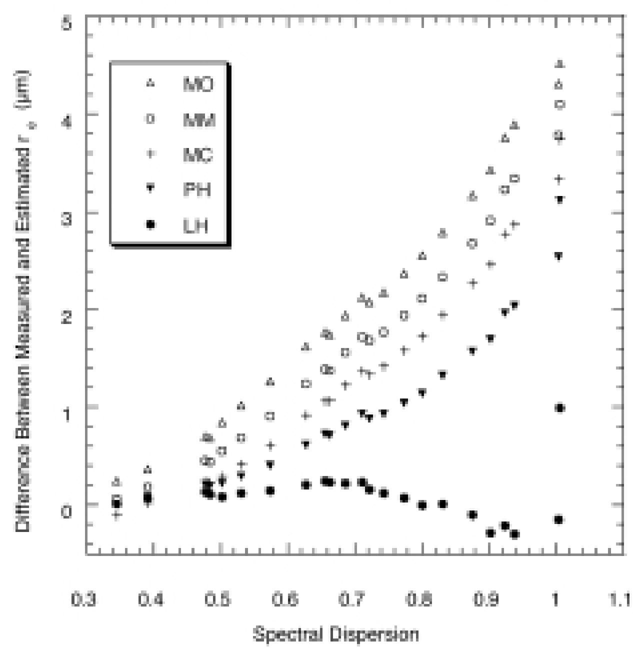

Figure 4. The difference between measured cloud droplet effective radius and those estimated from different parameterization schemes as a function of the spectral dispersion. Note the substantial reduction of errors by the Liu and Hallett scheme.