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WHICH SIZE DISTRIBUTION FUNCTION TO USE FOR STUDIES RELATED TO EFFECTIVE RADIUS

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Extended Abstract
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1. INTRODUTION

Effective radius r_e (defined as the ratio of the third to the second moment of a droplet size distribution) is one of the key variables that are used for calculation of the radiative properties of liquid water clouds (Hansen and Travis, 1974). The inclusion and parameterization of r_e in climate models has proven to be critical for assessing global climate change (Slingo, 1990; Dandin et al., 1997). There has been increasing evidence for parameterizing r_e as a 1/3 power law of the ratio of the cloud liquid water content (L) to the droplet concentration (N) (Pontikis and Hicks, 1992; Bower and Choularton, 1992; Bower et al. 1994; Martin et al., 1994; Liu and Hallett, 1997; Reid et al., 1998). The "1/3" power-law takes the form

$$r_e = \alpha \left(\frac{L}{N} \right)^{1/3} , \qquad (1)$$

where r_e is in μm, L in gm^{-3}, and N in cm^{-3}. The only difference among different power-laws lies in the specification of the prefactor α.

On the other hand, different analytical functions are often used to describe size distributions (Liu et al., 1995). From these analytical size distributions, we can derive "1/3" power-laws. In this work, existing expressions are compared and analyzed using the data collected during two Intensive Observation Periods (IOPs) conducted at the ARM (Atmospheric Radiation Measurements) program SGP (Southern Great Plain) site in Oklahoma, in the spring and fall of 1997.

2. EXPRESSIONS FOR α

For clouds with a monodisperse droplet size distribution as described by a delta function $n(r)=N\delta(r-r_e)$, $\alpha = 100(3/4\pi)^{1/3} \approx 62.04$; the multiplier 100 is introduced to keep the units of r_e, L and N in μm, g m^{-3} and cm^{-3}, respectively. This value of α was used by Bower and Choularton (1992), and Bower et al. (1994) to estimate the r_e of layer clouds and small cumuli. Martin et al. (1994) derived estimates of α of 66.83 for maritime, and 70.89 for continental stratocumulus clouds based upon analysis of in situ microphysical data. These expressions with fixed values of prefactor totally ignore the dependence of α on the spectral broadening processes. Pontikis and Hicks (1992) analytically derived an expression (PH) that relates α to the spectral dispersion d (defined as the ratio of the standard deviation to the mean radius of the corresponding droplet size distribution). Liu and Hallett (1997) derived another "1/3" power-law to allow for the effect of turbulent entrainment and mixing (WB) from the Weibull size distribution which itself derived from the systems theory proposed in Liu et al. (1995).

Besides the Weibull distribution, cloud droplet size distributions are often represented by the Gamma (GM) and lognormal (LN) distributions (Liu et al., 1995). More expressions can be easily derived from these two analytical size distributions. Table 1 summarizes all the prefactor expressions.

Table 1. Expressions for Prefactor α

MO	$\alpha = 62.04$
MM	$\alpha = 66.84$
MC	$\alpha = 70.91$
PH	$\alpha = 62.04 \frac{\left(1+3d^2\right)^{2/3}}{\left(1+d^2\right)}$
WB	$\alpha(b) = 64.52 \frac{\Gamma^{2/3}(3/b)}{\Gamma(2/b)} b^{1/3}$, $d = \left[\frac{2b\Gamma\left(2/b\right)}{\Gamma^2\left(1/b\right)} - 1 \right]^{1/2}$
GM	$\alpha = 62.04 \frac{\left(1+2d^2\right)^{2/3}}{\left(1+d^2\right)^{1/3}}$
LN	$\alpha = 52.04\left(1+d^2\right)$

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3. COMPARISON OF PREFACTORS

Figure 1 shows α as a function of d. Also shown in this figure are the α's for a monodisperse size distribution (MO), and Martin's values for continental (MC) and maritime clouds (MM). Substantial differences between these prefactor expressions are exhibited in Figure 1.

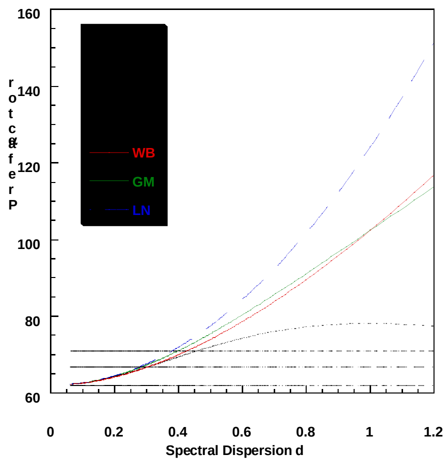

Figure 1. Prefactor α as a function of the spectral dispersion of the cloud droplet size distribution.

To address the question of their accuracy, these expressions are compared to those calculated from droplet size distributions collected with a FSSP during two recent IOPs at the ARM SGP site in northern Oklahoma in the spring and fall of 1997, respectively. During the two campaigns data from six flights in (broken) stratocumulus were analyzed. Figure 2 compares the measurements averaged over all the data sets with these different prefactor expressions, indicating that the WB and GM best fit the measurements, which are so close to each other that it is difficult to distinguish between them from these data sets. It is evident that the PH expression underestimates while the LN overestimates when droplet size distributions are broad, respectively. The PH, WB, GM and LN are almost equivalent for very narrow size distributions. The MO, MM and MC only represent cases with specific values of d.

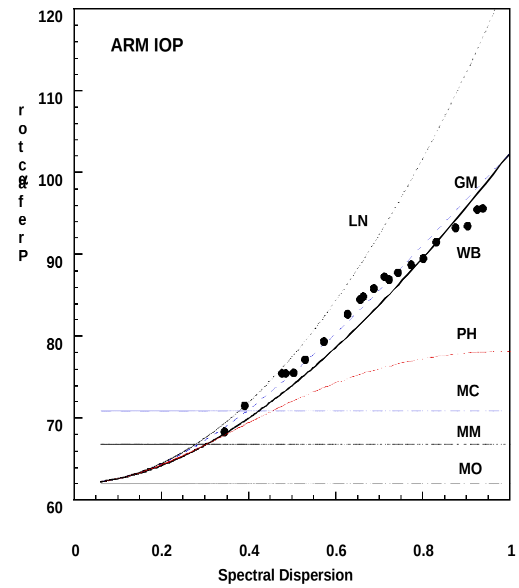

Figure 2. Comparison of prefactors calculated from the different expressions listed in Table 1 as a function of the spectral dispersion. The solid dots represent those derived from the FSSP-measured cloud droplet size distributions.

4. COMPARISON OF MEASURED AND PARAMETERIZED r_e

This section further illustrates the performance of these different parameterization schemes by comparing values of r_e measured by the FSSP (r_{em}) with those estimated from the different parameterization schemes. As indicated in Figure 3, the WB and GM schemes obviously outperform the others. The PH tends to underestimate while the LN tends to overestimate r_e.

It is expected from Eq. (1) that the differences in r_e estimated from the different parameterizations are due to the treatment of the dependence of α on d. This result can be better understood by examining the differences between r_{em} and parameterized r_e as a function of spectral dispersion. As shown in Figure 4, the bias of the estimated re from the measured values increases with d for all the parameterizations except for the WB and GM scheme. At large values of d, the bias of r_e could be larger than 2 μm, which is large enough to cause noticeable errors in climate models (Slingo, 1990).

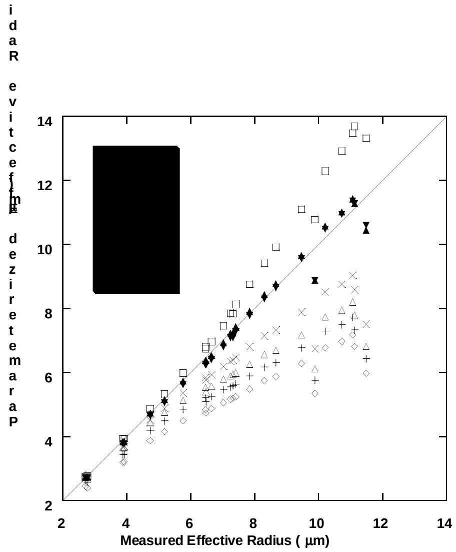

Figure 3. The cloud droplet effective radius estimated from the five different parameterization schemes as a function of the measured effective radius.

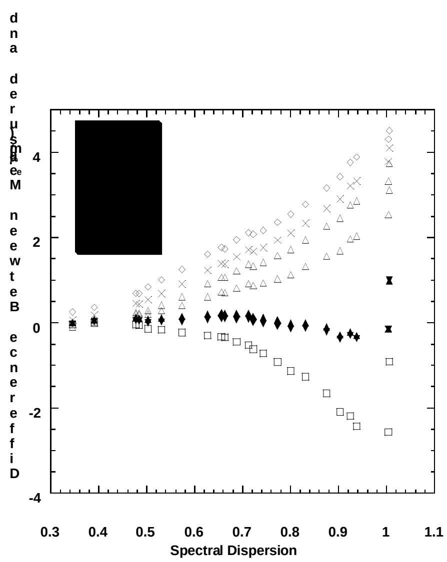

Figure 4. The difference between measured cloud droplet effective radius and those estimated from different parameterization schemes as a function of the spectral dispersion. Note the substantial reduction of errors by the WB and GM parameterizations.

5. WHY IS THE LH SCHEME MOST ACCURATE?

By a simple mathematical analysis, a universal "1/3" power-law can be derived (Martin et al., 1994; Liu and Yu, 1998)

$$r_e = 62.04\frac{\left(1+3d^2+sd^3\right)^{2/3}}{1+d^2}\left(\frac{\mathrm{L}}{\mathrm{N}}\right)^{1/3}, \qquad (2)$$

where s is the skewness of the size distributions. From Eq. (2), all the 7 schemes can be derived as special cases by substituting the corresponding "functions" between s and d. Therefore, to demonstrate why the WB and GM schemes parameterize r_e more accurately becomes to show that both schemes describe the s-d relationship more accurately. The result is evident from Figure 5.

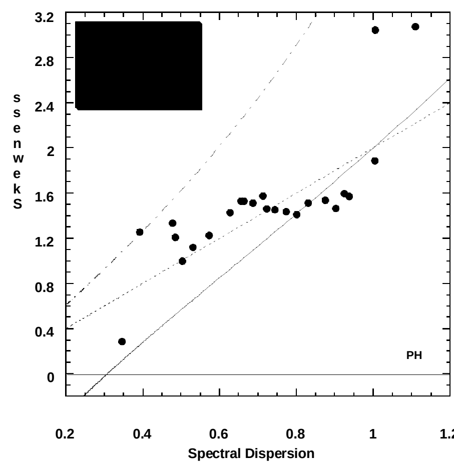

Figure 5. The relationship between the skewness and spectral dispersion. The dots represent the data points calculated from measured droplet size distributions.

6. CONCLUSIONS

Existing "1/3" power-law expressions for parameterizing r_e are compared and analyzed using data collected during two recent IOPs over the ARM SGP site. It is found that the Weibull-based and Gamma-based schemes most accurately represents the dependence of _ on d, and hence most accurately parameterizes r_e because of its accuracy in describing the dependence of s on d. It is also demonstrated that the bias of r_e by the parameterization schemes that have been widely used in current climate models could be large enough to cause serious problems. The result suggests that either Weibull distribution or Gamma distribution should be used to represent cloud droplet size distributions, particularly for studies related to the effective radius.

The state-of-art cloud parameterization in climate models is to predict L and N from which r_e is then determined using a "1/3" power-law with a fixed value of prefactor such as Martin's expression (Ghan et al., 1997; Lohmann et al., 1999). This study suggests that the prefactor is important as well. Accurately representing r_e in climate models requires predicting the prefactor in addition to liquid water content and droplet concentration. This study also suggests the convergence and consistence of microphysics parameterization in climate models with that in smaller-scale models such as cloud-resolving models: they also need to find the appropriate analytical size distribution for describing cloud droplet size distributions. In fact, finding an appropriate analytical size distribution itself is a fundamental change to cloud physics community.

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