

## Parameterization of Cloud Droplet Effective Radius: Effects of Spectral Dispersion and Skewness of Cloud Droplet Size Distributions

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### 1. INTRODUCTION

Effective radius  $r_e$  (defined as the ratio of the third to the second moment of a droplet size distribution) is one of the crucial variables that determine the radiative properties of liquid water clouds (Hansen and Travis 1974). The inclusion and parameterization of  $r_e$  in climate models has proven to be critical for assessing global climate change (Slingo 1990; Dandin et al, 1997). There has been increasing evidence for parameterizing  $r_e$  as a 1/3 power law of the ratio of the cloud liquid water content (L) to the droplet concentration (N) (Pontikis and Hicks, 1992; Bower and Choulaton 1992; Bower et al. 1994; Martin et al. 1994; Liu and Hallett 1997; Reid et al. 1998). The “1/3” power-law takes the form

$$r_e = \alpha \left( \frac{L}{N} \right)^{1/3}, \quad (1)$$

where  $r_e$  is in  $\mu\text{m}$ , L in  $\text{gm}^{-3}$ , and N in  $\text{cm}^{-3}$ . The only difference among different power-laws lies in the specification of the prefactor  $\alpha$ . In this work, existing expressions are compared and analyzed using the data collected during two Intensive Observation Periods (IOPs) conducted at the ARM (Atmospheric Radiation Measurements) program SGP (Southern Great Plain) site in Oklahoma, in the spring and fall of 1997.

### 2. EXPRESSIONS FOR $\alpha$

For clouds with a monodisperse droplet size distribution as described by a delta function  $n(r) = N\delta(r-r_e)$ ,  $\alpha = 100(3/4\pi)^{1/3} \approx 62.04$ ; the multiplier 100 is introduced to keep the units of  $r_e$ , L and N in  $\mu\text{m}$ ,  $\text{g m}^{-3}$  and  $\text{cm}^{-3}$ , respectively. This value of  $\alpha$  was used by Bower and Choulaton (1992), and Bower et al. (1994) to estimate the  $r_e$  of layer clouds and small cumuli. Martin et al. (1994) derived estimates of  $\alpha$  of 66.80 for maritime, and 70.91 for continental stratocumulus clouds based upon analysis of in situ microphysical data. These expressions with fixed values of prefactor totally ignore the dependence of  $\alpha$  on the spectral broadening processes. Pontikis and Hicks (1992) analytically derived an expression (PH and  $\alpha_{PH}$  hereafter) that relates  $\alpha$  to the spectral dispersion  $d$ , viz,

$$\alpha_{PH}(d) = 62.035 \frac{(1 + 3d^2)^{2/3}}{(1 + d^2)}, \quad (2)$$

where  $d$  is defined as the ratio of the standard deviation to the mean radius of the corresponding droplet size distribution. Liu and Hallett (1997) developed another “1/3” power-law from consideration of systems theory of. The resultant expression for  $\alpha$  (LH and  $\alpha_{LH}$  hereafter) is given by

$$\alpha_{LH}(b) = 64.52 \frac{\Gamma^{2/3}(3/b)}{\Gamma(2/b)} b^{1/3}, \quad (3)$$

where  $\Gamma(t) = \int_0^{\infty} z^{t-1} \exp(-z) dz$ . The parameter  $b$  depends on spectral broadening processes and relates to  $d$  by

$$d = \left[ \frac{2b\Gamma(2/b)}{\Gamma^2(1/b)} - 1 \right]^{1/2}. \quad (4)$$

### 3. COMPARISON OF PREFACTORS

Figure 1 shows  $\alpha_{PH}$  and  $\alpha_{LH}$  as a function of  $d$ . Also shown in this figure are the  $\alpha$ 's for a monodisperse size distribution (MO), and Martin's values for continental (MC) and maritime clouds (MM). Substantial differences between these prefactors are exhibited in Fig. 1. To address the question of their accuracy, these expressions are compared to those calculated from droplet size distributions collected with a FSSP during two recent IOPs at the ARM SGP site in northern Oklahoma in the spring and fall of 1997. During the two campaigns Data from six flights in (broken) stratocumulus were analyzed, and are displayed in Figure 2. This figure shows that  $\alpha_{LH}$  follows measured data points most closely.

### 4. COMPARISON OF MEASURED AND PARAMETERIZED $r_e$

This section further illustrates the superiority of the LH scheme by comparing values of  $r_e$  measured by the FSSP ( $r_{em}$ ) with those estimated from the different parameterization schemes. As indicated in Fig. 3, the LH scheme obviously outperforms the other schemes, which all underestimate  $r_e$  albeit to different degrees. This result can be better understood by examining the differences between  $r_{em}$  and parameterized  $r_e$  as a function of  $d$ . Figure 4 shows the increasing underestimation of  $r_e$  with  $d$  for all the schemes except for the LH scheme. At large values of  $d$ , the underestimation of  $r_e$  could be as much as  $3 \mu m$ , which is large enough to cause noticeable errors in climate models (Slingo 1990).

### 5. WHY IS THE LH SCHEME MOST ACCURATE?

By mathematical analysis, a universal "1/3" power-law can be derived (Martin et al., 1994; Liu and Yu 1998)

$$r_e = 62.04 \frac{(1 + 3d^2 + sd^3)^{2/3}}{1 + d^2} \left( \frac{L}{N} \right)^{1/3}, \quad (5)$$

where  $s$  is the skewness of the size distributions. From Eq. (5), all the five schemes can be derived as special cases by substituting the corresponding "functions" between  $s$  and  $d$ . Therefore, to demonstrate why the LH scheme parameterizes  $r_e$  most accurately becomes to show that the LH scheme describes the  $s$ - $d$  relationship more accurately than the PH scheme. The result is evident from Fig. 5.

### 6. CONCLUSIONS

Existing "1/3" power-law expressions for parameterizing  $r_e$  are compared and analyzed using data collected during two recent IOPs over the ARM SGP site. It is found that the LH scheme most accurately represents the dependence of  $\alpha$  on  $d$ , and hence most accurately parameterizes  $r_e$  because of its accuracy in describing the dependence of  $s$  on  $d$ . It is also

demonstrated that the underestimation of  $r_e$  by the parameterization schemes that have been widely used in current climate models could be large enough to cause serious problems.

The state-of-art cloud parameterization in climate models is to predict  $L$  and  $N$  from which  $r_e$  is then determined using a “1/3” power-law with a fixed value of prefactor such as Martin’s expression (Ghan et al. 1997; Lohmann et al. 1999). This study suggests that the prefactor is important as well. Accurately representing  $r_e$  in climate models requires predicting the prefactor in addition to liquid water content and droplet concentration.

## ACKNOWLEDGEMENTS

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## FIGURE CAPTIONS

**Fig. 1.** Dependency of the prefactor  $\alpha$  on the spectral dispersion of the cloud droplet size distribution. LH and PH refer respectively to the Liu and Hallett and the Pontikis and Hicks expressions. MC, MM and MO refer to Martin et al.'s values of  $\alpha$  for continental and marine clouds, and the value of  $\alpha$  for monodisperse size distributions, respectively.

**Fig. 2.** Comparison of prefactors calculated from the Liu and Hallett (solid curve) and Ponkitis and Hicks (dashed curve) expressions as a function of the spectral dispersion. The solid dots represent those derived from the FSSP-measured cloud droplet size distributions. The number on each plot such as “970420a” denotes flight numbers.

**Fig. 3.** The cloud droplet effective radius estimated from the five different parameterization schemes as a function of the measured effective radius. LH, PH, MC, MM, and MO represent the effective radius estimated from the corresponding parameterization schemes, respectively. Note the improvement of the Liu and Hallett scheme over the others, especially for  $r_{em} > 8 \mu\text{m}$ .

**Fig. 4.** The difference between measured cloud droplet effective radius and those estimated from different parameterization schemes as a function of the spectral dispersion. Note the substantial reduction of errors by the Liu and Hallett scheme.

**Fig. 5.** The relationship between the skewness and spectral dispersion. The dots represent the data points calculated from measured droplet size distributions.

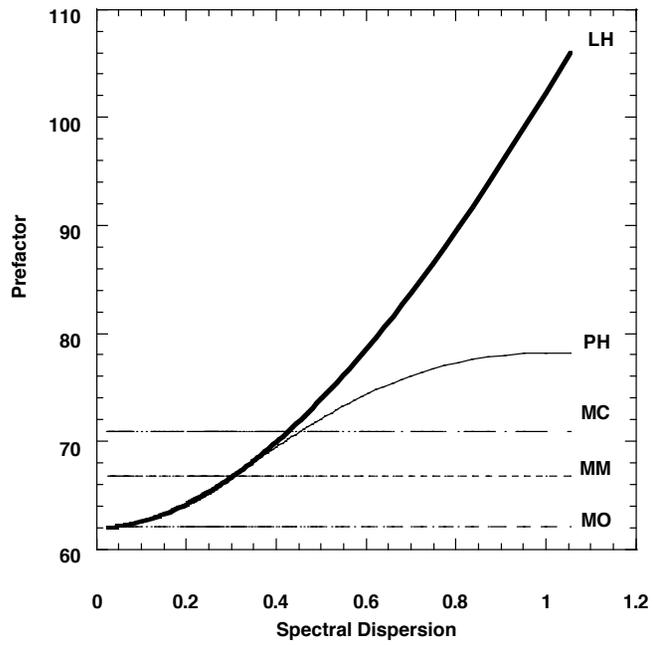


Figure 1

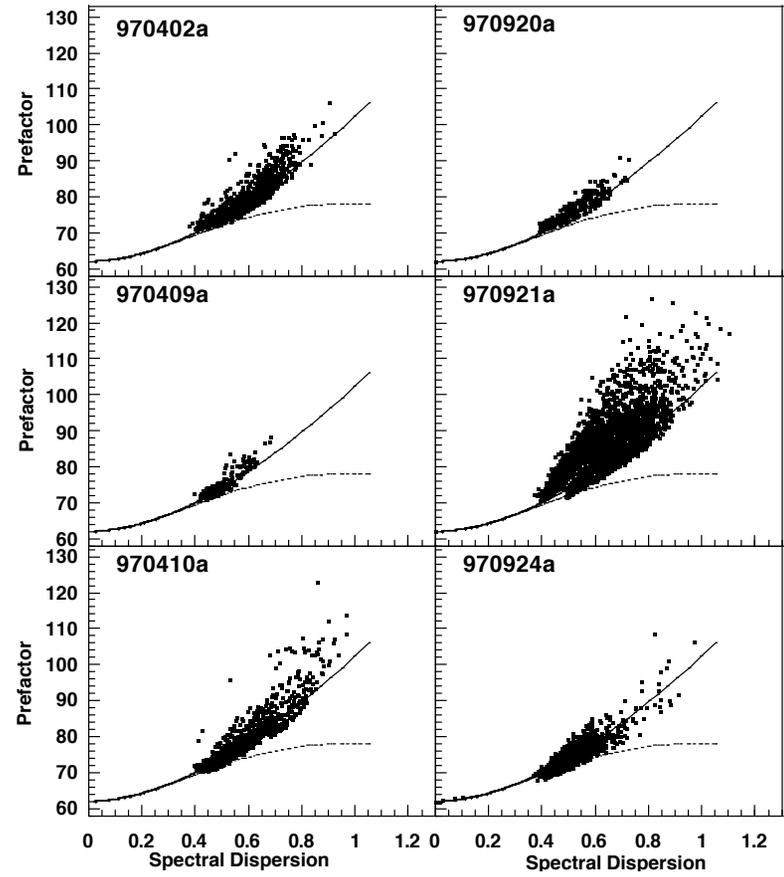


Figure 2

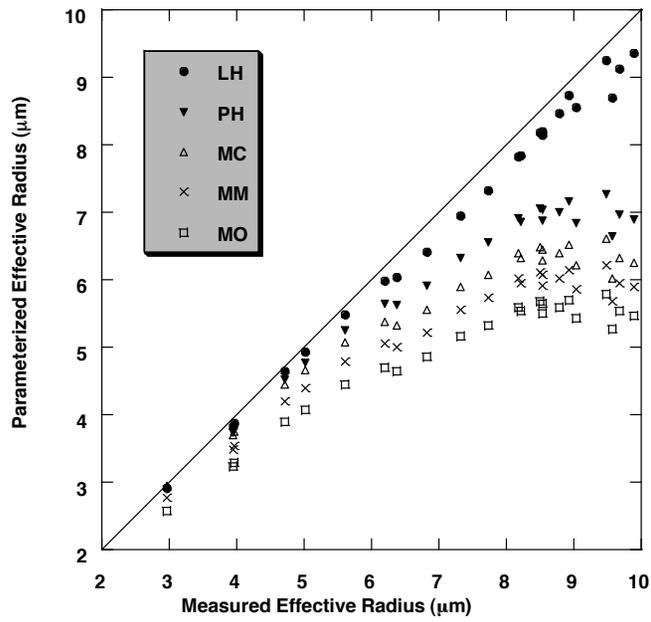


Figure 3

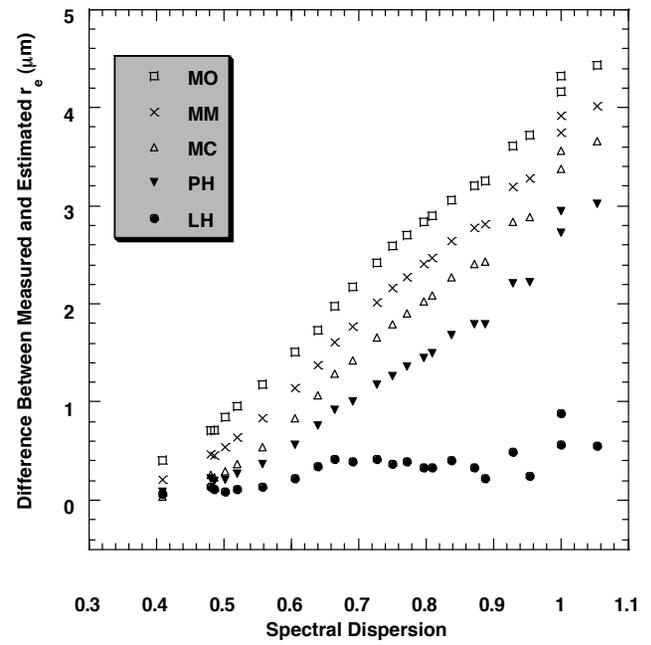


Figure 4

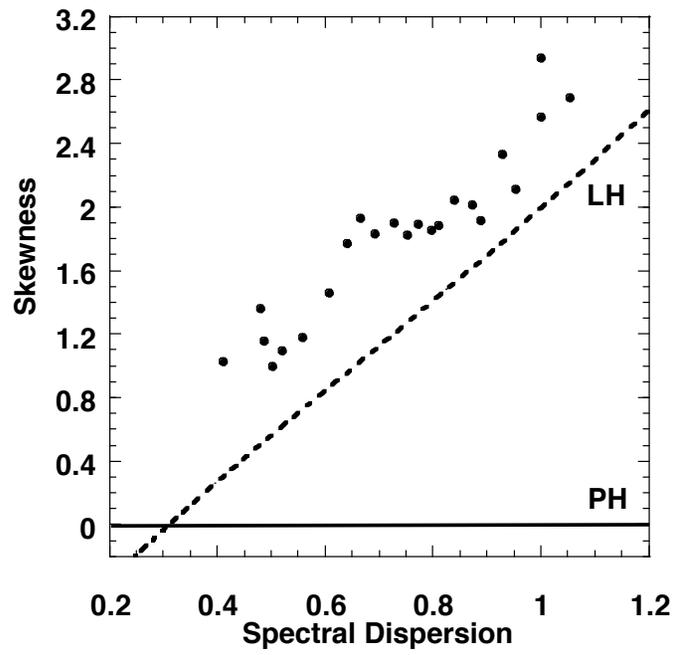


Figure 5