Response to Comments on
“Heat capacity, time constant, and sensitivity of Earth's climate system”

Stephen E. Schwartz

Atmospheric Science Division, Brookhaven National Laboratory

Upton NY 11973

ses@bnl.gov

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Abstract

Reanalysis of the autocorrelation of global mean surface temperature prompted by the several Comments, taking into account a subannual autocorrelation of about 0.4 year and bias in the autocorrelation resulting from the short duration of the time series has resulted in an upward revision of the climate system time constant determined in Schwartz [2007] by roughly 70%, to 8.5 ± 2.5 years (all uncertainties are 1-sigma estimates). This results in a like upward revision of the climate sensitivity determined in that paper, to 0.51 ± 0.26 K/(W m^{-2}), corresponding to an equilibrium temperature increase for doubled CO_2 of 1.9 ± 1.0 K, somewhat lower than the central estimate of the sensitivity given in the 2007 assessment report of the Intergovernmental Panel on Climate Change, but consistent within the uncertainties of both estimates. The conclusion that global mean surface temperature is in near equilibrium with the applied forcing continues to hold. Forcing over the twentieth century other than that due to greenhouse gases, ascribed mainly to tropospheric aerosols, is estimated as -1.1 ± 0.7 W m^{-2}.

Introduction

Foster et al [2008, hereinafter FASM], Knutti et al. [2008, hereinafter KKFA], and Scafetta [2008, hereinafter NS08] have all raised important questions with respect to my paper "Heat capacity, time constant, and sensitivity of Earth's climate system [Schwartz, 2007; hereinafter S07]. I am pleased to have the opportunity to respond to these questions, to present a reanalysis of the data stimulated by the several Comments, and to elaborate on some of the assumptions underlying the analysis presented in S07.

It would seem that much of the criticism of S07 arises from the relatively low climate sensitivity that resulted from that analysis, 0.30 ± 0.14 K/(W m^{-2}), corresponding to an equilibrium temperature increase for doubled CO_2 $\Delta T_{2\times}$ = 1.1 ± 0.5 K, considerably lower than the best estimate and associated likely uncertainty range for this quantity given by the 2007 assessment by the Intergovernmental Panel on Climate Change [IPCC, 2007], 2 – 4.5 K, > 66% likelihood.

S07 explicitly noted several possible areas of concern with the analysis presented in that paper:

1. The effective heat capacity that is coupled to the climate system, as determined from trends in ocean heat content and global mean surface temperature GMST, 17 ± 7 W a m^{-2} K^{-1} (all uncertainties are 1-sigma estimates; the symbol a, for annum, is used for the unit year) might too low, or too high. For climate sensitivity λ_s^{-1} related to global heat capacity C and climate system time constant τ as

$$\lambda_s^{-1} = \tau / C. \tag{1}$$

a value of heat capacity that is too great would result in an erroneously low climate sensitivity.

2. The method of empirically inferring the climate system time constant τ, analysis of temporal autocorrelation GMST, might not yield an accurate estimate of this quantity that is pertinent to climate change on the decadal to centennial time scale. An erroneously low value of τ would result in an erroneously low climate sensitivity.

3. Earth's climate system is too complex to be accurately represented by a single compartment energy balance model.

The several Comments pick up on these points and others. This response deals first with questions regarding the energy balance model and then turns to details of the quantitative interpretation of the observational data.

Energy balance model

The model employed by S07 consisted of a planetary energy balance model such that the change in planetary heat content with time dH/dt due to an imbalance between absorbed shortwave power Q and emitted longwave power E is represented by a change in global mean surface temperature with time dT_s/dt times an effective heat capacity C.

$$\frac{dH}{dt} = Q - E = C\frac{dT_s}{dt} \tag{2}$$

According to this model the equilibrium climate sensitivity of the planet, λ_s^{-1}, the equilibrium change in GMST per change in long- or shortwave flux, is related to the effective heat capacity by Eq (1). A key concern with this model raised by FASM is that Earth's climate system consists of numerous components that would exhibit a multiplicity of heat capacities which would lead it to exhibit numerous time scales. In particular they call attention to the deep ocean component which would exhibit a much longer greater characteristic time than the upper ocean, which they imply is the component which is dominating the short time constant found in S07. In particular they assert that "the principal physical mechanism which leads us to believe that not all committed greenhouse gas warming has yet been experienced, and a substantial amount remains 'in the pipeline,' is the warming of the deep ocean [Hansen et al., 2005]."

Earth's climate system, more specifically, global mean surface temperature GMST, would certainly be expected to exhibit numerous time scales, from subannual, to multidecadal (as was the objective of the examination of S07), to millennial and beyond. An underlying assumption of S07 is that the heat reservoir giving rise to the heat capacity exhibiting the multidecadal time scale found in that study is sufficiently decoupled from other heat reservoirs having much longer time constants that its time constant is meaningful and can be determined from the autocorrelation of GMST on the century time scale.

The basis of the analysis of S07 and the applicability of this analysis in situations of multiple heat capacities may perhaps be heuristically conveyed by analogy to an equivalent electrical circuit, Figure 1. Consider first circuit *a*. It is desired to determine the sensitivity of the voltage V of a circuit initially in steady state to an increment ΔI in incoming current I. This sensitivity $S = \Delta V / \Delta I$ is equal to the resistance R, which is not known. Assume however that it is possible to measure the voltage V and also the change in the charge Q on the capacitor C under circumstances in which the voltage is increasing, for example by measuring the current i into the capacitor. Then the capacitance can be determined as $C = dQ / dV = (dQ / dt) / (dV / dt) = i / (dV / dt)$.

Assume further that there are fluctuations in the incoming current I, such that the time constant of the system $\tau = RC$ can be determined from autocorrelation analysis of the fluctuations in the voltage V. Then the (unknown) resistance can be determined as $R = \tau / C$. That in essence is the basis of the analysis of S07.

The electrical circuit analogy helps to demonstrate how the short time constant determined in S07 can be pertinent to the determination of climate sensitivity even when there are other contributions to global heat capacity. Consider an additional large capacitance C' that is weakly coupled to the initial circuit by a large resistance R'. This additional circuit element has its own time constant $\tau' = R'C'$. The overall circuit will be characterized by two time constants (inverses of the eigenvalues); for $\tau' >> \tau$ the two time constants are approximately τ and τ', respectively. Importantly the equilibrium sensitivities of the two circuits are the same, the difference being that circuit *b* requires a much greater time to reach equilibrium than circuit *a*.

The electrical circuit analogy has further value in interpreting time behavior of the change in global mean temperature that would result from a sustained forcing. Consider circuit response to a step function forcing ΔI. At times greater than τ but less than τ' the voltage V would exhibit an apparent equilibrium value which would be less than the true equilibrium value because the current flowing across the resistor R would be diminished by the current flowing into capacitor C'. The difference between the apparent equilibrium voltage and the true equilibrium voltage, which would become manifested on the longer time scale τ', might be considered an additional voltage that is "in the pipeline."

The electrical circuit analogy also points to a means of estimating the longer time constant τ'. Consider a sustained forcing ΔI applied to the system initially at steady state. Suppose that the magnitude of the second, large capacitance C' is known and that it is possible to determine the current i' into that capacitance at some time short compared to the time constant τ' of the second circuit but at a time sufficiently long that the first circuit has reached its steady state; that is, at a time well greater than τ. Measurement of this initial current i' permits determination of the time constant τ' as the quotient of the charge Q' that the capacitance will hold when charged to the voltage V, i.e., $Q' = VC'$, divided by the current i'; that is, $\tau' = Q'/i' = VC'/i'$.

This analogy can be applied directly to obtain an estimate of the time constant associated with the larger heat capacity C' that deep ocean water contributes to Earth's climate system as

$$\tau' = \frac{C'\Delta T}{dH'/dt} \tag{3}$$

where C' is the heat capacity of the deep ocean, dH'/dt is the rate of increase of the heat content in this reservoir, and ΔT is the temperature increase driving that heat transfer. For a global average ocean depth of 3800 m, ocean fraction of global surface area 0.71, and volume heat capacity of seawater taken as 4×10^6 J m^{-3} the global mean areal heat capacity of the deep ocean is 3.4 W a m^{-2} K^{-1}. This heat capacity is 25 times that of the ocean that is coupled to the climate system as determined in S07. The time constant associated with this heat capacity is evaluated taking the temperature increase of the climate system over the industrial period as 1 K and the heat flow into the deep ocean as 0.1 W m^{-2}, the latter as estimated from coupled ocean-atmosphere model calculations by Hansen et al. [2005]; the exact values of these quantities of no consequence for the purpose of this scoping calculation. The resultant time constant is about 3000 years, well longer than that of the climate system determined in S07 as 5 a (or than the revised value given below of 8.5 a). Thus the two "circuits" may be considered essentially decoupled, justifying the evaluation of the climate system time constant in S07 as uncoupled from other contributions to global heat capacity.

A more precise estimate of the fraction of global heat uptake that is going into the deep ocean is necessary to evaluate the additional heating that is "in the pipeline" and that would be expressed as the deep ocean equilibrates, over the millennial time scale, to the warming of the small fraction of the world ocean that is coupled to the climate system on the multidecadal time scale. This coupling would, for constantly maintained forcing, increase the global mean surface temperature by an additional amount equal to the fraction of heat now going into the deep ocean, 17%, by the above estimate. This incremental temperature increase would be expressed on the time scale of the larger heat reservoir, that is, 3000 years.

Finally it should be stressed that although the amount of heat capacity that is coupled to the climate system is equivalent to only 110 m of seawater, or for ocean fractional area 0.71, 150 m of ocean depth, this heat capacity is fairly deeply distributed, with more than half of the total heat capacity (to the 3000 m in the data compilation of Levitus et al. [2005] below 300 m (Table 2 of S07). The heat uptake is not uniform, as might be expected for diffusive transport across the thermocline from the mixed layer to the deep ocean, but rather is spatially quite heterogeneous as a consequence of transport in descending plumes associated with deep water formation [Levitus et al., 2005, Figure 2] especially in the North and South Atlantic oceans [Barnett et al., 2005].

Empirically determined heat capacity

KKFA raise questions over the accuracy of the data in the Levitus compilation used in determination of the heat capacity pertinent to climate change on the multidecadal scale by S07, noting concerns over instrument calibration, changes in instrument types over time, poor sampling coverage and interpolation schemes and citing in support of those concerns Gregory, et al. [2004] and AchutaRao, et al. [2006] among others. They state further that "the decadal variations in ocean heat uptake are poorly understood, not well simulated in models, and may be partly caused by interpolation of the sparse data, again citing Gregory, et al. [2004] and AchutaRao, et al. [2006].

While the accuracy of measurements is always a legitimate avenue of concern, it is questionable whether measurements should be rejected because they do not agree with models, especially such complex models as global climate models, which are based on many parameterizations and which differ in important ways among each other and from many observables. In fact, however, the accuracy and utility of the data in the Levitus et al. compilation have received support in much other work. Barnett et al. [2005], whose authors include several of the investigators in the papers cited by KKFA, lend strong support to the accuracy and utility of the data in the Levitus et al compilation, characterizing that data set as "the best available description of the ocean's warming signal and its evolution through time." Barnett et al. [2005] conclude as well that the warming signal which has penetrated into the world's oceans over the past 40 years "is well simulated by two anthropogenically forced climate models." That study also states that the conclusion reached therein, that a warming signal has penetrated into the world's oceans over the past 40 years is "robust to observational sampling". In support of the latter statement Barnett et al. state that although they had used a sampling strategy that compares model and observations only where observations exist, not using infilled or interpolated data set, as a test, they repeated the analysis using the infilled data and found that it made no difference to the conclusions.

As KKFA note, errors from instrument calibration and changes in instrument types are inevitably a concern in using observational data, especially data from long time series. With respect to the accuracy of the ocean temperature data that are the basis of the Levitus et al. [2005] analysis, Ingleby and Huddleston [2007] in introducing a thorough quality-controlled analysis of these data found, on the basis of detailed examination of paired data and the like, minimal consequences of measurement error in the data.

Empirically determined climate system time constant

Internal versus external forcing. FASM draw the distinction between variability in GMST that arises from processes that are internal to the climate system versus that arising from climate system response to external forcings, stating that it is unlikely that analysis of fluctuations that arise from external forcings can be described, as was done in S07, as a Markov or AR(1) process. Here it may be recalled that Einstein's examination of the motion of a brownian particle that identified the intrinsic relation between the relaxation time constant of a system and its temporal autocorrelation was explicitly an examination of the particle's response to random external forcing (molecular collisions) rather than any response to internal processes. It would thus certainly seem that the fact that much of the short term variation in GMST over the instrumental record is reflective of climate system response to random "external" forcings such as volcanic eruptions should be taken not as an argument against the pertinence of these fluctuations to inferring climate system time constant, but rather as supportive of that approach.

Multiple time constants. All three Comments raise concern over the possibility of multiple time constants that characterize Earth's climate system whose existence would invalidate the interpretation of autocorrelation of GMST under the assumption of a linear trend plus a first-order Markov process and the inference of a time constant from this autocorrelation. As explicitly noted in S07 values of $\tau(\Delta t)$ evaluated as

$$\tau(\Delta t) = \frac{-\Delta t}{\ln r(\Delta t)}, \tag{4}$$

where $r(\Delta t)$ is the autocorrelation as a function of lag time Δt , were found to increase with increasing lag time from about 2 years at lag time 1 a, reaching an asymptotic value of about 5 years by about lag time Δt = 8 yr. FASM argue this increase reveals a shorter time constant whose existence invalidates the assumption of S07 that the GMST data can be interpreted as an AR(1) process.

In S07 the value of the climate system time constant was evaluated, by visual inspection of the plot of $\tau(\Delta t)$ vs. Δt, as 5 ± 1 a. In his Comment Scafetta [NS08] proposes an alternative method of determining the characteristic time constant, again from the time dependence of the autocorrelation, as

$$\tau = -\frac{1}{d\ln r(\Delta t)\,/\,d\Delta t} \tag{5}$$

the negative inverse of the slope of the graph of $\ln r$ versus Δt, rather than by visual inspection. This approach has the advantage of yielding a more objective value for τ that uses all the data and of yielding an asymptotic value of τ. Applying this approach to the monthly values of GMST, Scafetta found it necessary to represent the data with two time constants, one characterizing the decorrelation on the time scale of 0 to 2 years that exhibits a slope corresponding to a time constant of 0.40 ± 0.1 a, (~5 months) and a second one pertinent to the decorrelation on a time scale up to at least 20 years whose slope corresponds to a time constant of 8.7 ± 2 a. I would assert that the existence of the short time scale is irrelevant to the interpretation of climate change on the multidecadal time scale, which was the objective of S07, but that it confounds the interpretation of the data as an AR(1) process. In particular, as clearly shown in the semi-logarithmic plots presented by NS08, the autocorrelation data are not at all well fit by the single time constant (5 ± 1 a) advanced by S07 nor by any single time constant.

In retrospect the existence of autocorrelation on a time scale of months, even in global mean surface temperature, should not be considered surprising, likely being reflective of persistence of weather patterns or the like. But such short term autocorrelation is of no consequence to considerations of climate change on the multidecadal time scale, other than raising question over the applicability of the interpretation of the autocorrelation on the longer time scale as a Markov process. More specifically such short-term autocorrelation and the resultant departure of autocorrelation from AR(1) behavior should not be advanced as an argument against inferring the climate system time constant pertinent to the century-long observational data from the autocorrelation at longer time scales and should in no way invalidate the interpretation of S07 that the asymptotic approach of τ to a constant value at lag times as great as 15-18 years suggests that the time constant obtained in this way is reflective of the time constant of the climate system on a multi-decadal scale pertinent to changes over the industrial period.

Stimulated by Scafetta's Comment [NS08] I present in Figure 2 a semilogarithmic plot of the autocorrelation coefficient versus lag time for the deseasonalized monthly average global mean surface temperatures from the GISS meteorological station data set as examined in S07. In view of the rapid decrease in autocorrelation within lag times of 1-2 years it seems advantageous to confine further examination of these autocorrelations to the monthly data rather than the annual data; the use of the monthly data also provides many more independent measurements, lending enhanced confidence to the results. Similar plots were constructed for the Northern Hemisphere and Southern Hemisphere meteorological station data, for the GISS global land-ocean data set, and also for the CRU (HadCRUT3) global and hemispheric data sets. According to the two time-constant model advanced by NS08 the longer time constant, which is the quantity of interest here, can be accurately obtained by Eq (5) from the slope of a semi-logarithmic graph of $r(\Delta t)$ vs. Δt at lag time Δt sufficiently great that the short-time-constant autocorrelation is negligible, that is greater than about 3 years. In carrying out fits to the data pertinent to the longer time constant of interest here, only the data for lag time Δt = (4, 11) a were used, to avoid the influence of a greater autocorrelation at short lag times noted by Scafetta. The upper limit of the fit range was selected by inspection of the plot to avoid the great increase in uncertainty in $\ln r$ as r approaches 0 at large Δt. In any event the slope is not greatly sensitive to the choice of the limits of the fit. The limiting value τ of the climate system time constant for large Δt was evaluated for each of the data sets as the negative inverse of the slope of a linear fit of $\ln r(\Delta t)$ vs. Δt as summarized in Table 1. For the GISS global data set the time constant obtained in this way is 8.6 ± 0.7 a; comparable or somewhat lower values were obtained with the CRU global data set and with the hemispheric data sets. The values of τ thus obtained, which are systematically greater than the estimate given in S07, are much more likely to be representative of the time constant pertinent to climate change on the multidecadal scale, as suggested by Scafetta.

An alternative approach to examination of the data results from recognition that the rapid decrease in autocorrelation at short lag time Δt is due to the short time constant, which is not of interest from the perspective of determining climate sensitivity on the multidecadal time scale. Specifically the $\Delta t = 0$

intercept of the linear fit of $\ln r(\Delta t)$ vs. Δt, $\ln r_0$ (Figure 2*a*), yields a value of autocorrelation r_0 that represents the decrease in autocorrelation due to the short time constant; once the effect of the rapid time constant has decayed away, the residual autocorrelation is given as $r(\Delta t) = r_0 \exp(-\Delta t / \tau)$. If this decrease is accounted for, the remaining autocorrelation is due to the longer time constant; permitting evaluation of this time constant as a function of lag time as

$$\tau(\Delta t) = \frac{-\Delta t}{\ln r(\Delta t) - \ln r_0} \qquad (6)$$

in lieu of Eq (4). The time constant $\tau(\Delta t)$ evaluated by Eq (6) is presented in Figure 2*b* as a function of lag time Δt (green points) along with the values obtained with Eq(4) as given by S07 (red points). This procedure yields values of τ that are essentially independent of lag time and scattered about the value obtained from the slope (horizontal green line), rather than slowly asymptotically approaching this value when the rapid decay in autocorrelation due to the short time constant is not accounted for. It is clear from this graph that the approach of S07, which yielded an estimate of τ of 5 ± 1 a resulted in an underestimate of this quantity and that a more accurate estimate of this quantity would be about 9 a for the GISS global data. Somewhat shorter time constants were obtained with the CRU data set, comparable to but somewhat lower than the result presented by Scafetta for the CRU data set, 8.7 ± 2 a.

To more directly compare the present approach with that of Scafetta [NS08] I explicitly fit the observed autocorrelation data to his expression for two time constants, $r(\Delta t) = A \exp(-\Delta t / \tau_1) + (1 - A) \exp(-\Delta t / \tau_2)$, shown as the dashed red curve in Figure 2. As anticipated, for intermediate lag times the values of the longer time constant, τ_2 resulting from this approach, Table 1, are comparable to the results obtained by the single slope approach.

In sum, it is clear that the estimate of the climate system time constant given by S07 based on visual inspection of the time constants evaluated for individual autocorrelation times (Eq 4), 5 ± 1 a, is erroneously low on account of the influence of a shorter time constant which results in a rapid decrease in autocorrelation at time scales up to 2-3 a and which therefore results in an inordinately long lag time until the time constant approaches its asymptotic value. Accounting for the influence of the shorter time constant results in the time constant being longer than that given by S07, 8.8 ± 2 a for the GISS GMST data set; 7.2 ± 1.5 a for the CRU GMST data set, where the uncertainties are intended to encompass the values obtained by the several approaches.

Bias from shortness of the data record. All three Comments raise concern over bias in the inferred autocorrelation coefficient due to the short record of observational data; S07 used tine series of GMST from 1880 through 2004. The concern is that the period of record (125 years as used in S07) is not sufficiently greater than the inferred time constant (5 a in S07 or 8 a above) that the resulting inferred autocorrelation coefficient is free of bias due to the shortness of the time series; the bias would be all the greater for a larger time constant. The bias would lead to an autocorrelation that falls off too quickly with increasing lag time and in turn to too short an inferred climate system time constant. NS08 presents a comparison of time constant inferred from synthetic data having a time constant of 12 a; the value obtained from a time series of 125 a, 8.2 a, was much shorter than that obtained with a time series of 1500 a. FASM and KKFA note similar concerns, This concern is well taken and therefore invites further examination.

There is no universally accepted method for estimating or removing bias from estimates of autocorrelation of time series, and a variety of alternative method have been advanced [Quenouille, 1949; Marriott and Pope, 1954; Kendall, 1954; Huitema and McKean, 1991] in addition to the method of Tjostheim and Paulsen [1983] cited by FASM. The method of Quenouille offers an empirical means of determining and correcting for autocorrelation in a time series by evaluation of the autocorrelation coefficients from the first and second halves of the time series, r_1 and r_2, in addition to that for the time series as a whole; an unbiased estimate of the autocorrelation coefficient obtained from consideration of the reduction in autocorrelation in the two halves of the time series relative to the series as a whole is given as

$$r_{\mathrm{u}} = 2r - (r_1 + r_2)/2 \tag{7}$$

As shown by Marriott and Pope [1954] this procedure reduces the bias in the autocorrelation coefficient to order N^{-2}; those investigators note also that in contrast to other methods, this method does not rely on any assumption about the nature of the autocorrelation characterizing the time series. It has the further advantage of yielding unbiased estimates of the autocorrelation coefficient for all time lags Δt. A concern with this method is that it can yield autocorrelation coefficients that are greater than unity when the autocorrelation coefficients in the two halves of the time series differ for reasons other than the length of the time series.

The unbiased estimates of the autocorrelation coefficients $r_{\mathrm{u}}(\Delta t)$ determined according to Eq(7) for each value of lag time Δt for the raw autocorrelation data obtained from the time series of GMST for the GISS and CRU data sets are shown in blue in Figure 2*a*. As expected, these unbiased estimates are systematically greater than those calculated without accounting for bias. As also with the uncorrected data the time constant calculated by Eq (6) shows little systematic dependence on Δt for $\Delta t \gtrsim 4$ a, indicative that the effect of the short time constant has been accounted for.

In summary, the correction for bias due to the shortness of the time series was found increase the time constant inferred from the GISS GMST data set by 5 to 25 %, depending on the approach; for the CRU GMST data set the bias estimate actually led to a slight reduction in the estimated time constant. These findings suggest that bias due to the shortness of the time series is slight.

Revised estimate of climate system time constant. Consideration of the consequences of the presence of a subannual time constant in addition to the longer time constant of concern here and the bias due to the shortness of the time series leads to an upward revision of the climate system time constant as determined from the autocorrelation of GMST from the value of 5 ± 1 a given in S07 to 8.5 ± 2.5 a, where, again, the uncertainty is meant to encompass the determinations by the several methods for the two data sets. The implications of this upward revision of the climate system time constant on other derived quantities are examined below.

Empirically determined climate sensitivity

Treatment of uncertainties. KKFA express concern that the estimates of uncertainty in climate sensitivity in S07 are too low, especially as sensitivity λ_s^{-1} is evaluated, Eq. (1), as the quotient of two quantities, time constant τ and heat capacity C, both of which have large relative uncertainties. In particular they suggest that the large uncertainty in the denominator of (1) together with an assumed normal distribution would lead to a skewed distribution with a large positive tail that is not properly accounted for in S07.

In response it must be emphasized that the intent of S07 in characterizing the estimates of uncertainty in τ and C as "1 sigma" was not to imply a normal distribution but simply to give a sense of the meaning of the estimated uncertainty; as was stated clearly in S07 the uncertainties were estimated from the spread of the results for the several data sets examined and several approaches to infer heat capacity and time constant. In estimating the uncertainty in λ_s^{-1} the uncertainties in τ and C were propagated in the conventional manner for uncorrelated quantities (e.g. Bevington [1969]), that is, the fractional uncertainty in a quotient is evaluated as the square root of the sum of the squares of the fractional uncertainties in numerator and denominator. As for the uncertainty in C, being in a denominator, giving rise to a skewed distribution with a long positive tail, it should be remarked that the determination of C from the regression of ocean heat content versus temperature in S07 by the ordinary least squares bisector method in S07 treated both variables symmetrically in the least squares analysis; one might thus equally well have expressed the result of that determination as an inverse heat capacity C^{-1} with identical fractional uncertainty, but which, in the evaluation of the climate sensitivity, would have entered into a product rather than a quotient, and which would therefore not give rise to a skewed distribution with a long positive tail.

Revised determination of climate sensitivity. The upward revision of the climate system time constant by approximately 70% results, by Eq (1), in a like upward increase in the value of the climate sensitivity from the value given in S07, 0.30 ± 0.14 K/(W m^{-2}) to 0.51 ± 0.26 K/(W m^{-2}), corresponding for the forcing of doubled CO_2 taken as 3.7 W m^{-2}, to an equilibrium increase in GMST for doubled CO_2 $\Delta T_{2\times}$ of 1.9 ± 1.0 K. Although this value is still rather low compared to many current estimates it is much more consistent than the value given in S07 with the estimate given in the Fourth Assessment Report of the IPCC [2007] as "2 to 4.5 K with a best estimate of about 3 K and ... very unlikely to be less than 1.5 K".

Implications on other inferred properties of the climate system. As pointed out in S07, once the climate sensitivity is known it is possible to infer total forcing over a specific period from the observed change in GMST over that period as an "inverse calculation" [Anderson et al., 2003]. The revision in estimated climate sensitivity relative to that of S07 results in a revision of Table 3 of that paper in which total forcing and forcing other than by greenhouse gases were presented; that revision is shown here as Table 2. Perhaps most important here is the revision in the forcing other than by greenhouse gases, which is attributed mainly to forcing by anthropogenic aerosols, which is given now as -1.1 ± 0.7 W m^{-2}, substantially greater (negative) forcing than given in S07. The conclusion of S07 that changes in atmospheric composition over the industrial period would, for concentrations of forcing agents held constant at present values, lead to minimal additional heating "in the pipeline" is unchanged.

Comparisons with climate models

The results of application of the diagnostic approach of S07 to examination of the time series of GMST and net planetary heat uptake calculated with GCMs, as presented by FASM and KKFA, are disquieting, particularly the large differences exhibited between the analyses of model results versus observational data. Certainly, if the models accurately represent the processes that govern various climate observables, these quantities should exhibit properties similar to those characterizing Earth's climate system as derived from observation. Here attention is called again to the study of Wigley et al. [1998], which compared the autocorrelation spectra of two GCMs with observations, concluding that differences in the autocorrelation in the twentieth century observational data from those of unforced model runs could be taken as evidence of externally forced climate change over the twentieth century.

Given the major differences between the results obtained by applying the approach of S07 to observed and modeled climate data, the question arises as to the reason or reasons for this. Several possible reasons might be advanced for the major discrepancies between application of the approach of S07 to observed and modeled climate data:

1. Errors and uncertainties in the observations and, especially for ocean heat content data, limited extent and duration of measurements.

2. Shortness of the time record of the observations, precluding statistically meaningful inferences especially of the autocorrelation.

3. Inherent flaws in the approach to the inference of climate system time constant, from autocorrelation analysis, due to the complexity of the climate system and a multiplicity of time constants characterizing the climate system that precludes the applicability of such a simple relation as Eq (1) to determine climate sensitivity.

4. Inaccuracy in modeled quantities that serve as the basis for comparison with observations.

While these possible explanations cannot be fully examined here, some conclusions can be drawn that might usefully point the way to future analyses. The extension of S07 by Scafetta [NS08] already limits the utility of examining application of the method of S07 to determination of the climate system time constant and sensitivity as presented in the Comments of FASM and KKFA.

Heat capacity. Both FASM and KKFA present values of this planetary heat capacity inferred from slopes and/or correlations of time series of net planetary heat uptake and GMST from coupled ocean-atmosphere global climate models. KKFA reported heat capacities inferred from the output of 17 three dimensional coupled atmosphere ocean general circulation models (AOGCMs) which participated in the World Climate Research Programme's Coupled Model Intercomparison Project phase 3 multi-model dataset and which they characterize as providing the most comprehensive available description of the climate system. While KKFA characterize the average heat capacity inferred from the output of those models, 24 W a m^{-2} K^{-1} as in "reasonable agreement" with the estimate of S07, 17 ± 7 W a m^{-2} K^{-1}, the range, 7 to 45 W a m^{-2} K^{-1} (a factor of 6) and the relative standard deviation 0.48 are quite large. Even different models from the same groups yielded quite different heat capacities: 11 W a m^{-2} K^{-1} for the GISS-EH model vs. 41 W a m^{-2} K^{-1} for the GISS-ER model; 7 W a m^{-2} K^{-1} for the HADGEM1 model vs. 24 W a m^{-2} K^{-1} for the HADCM3 model (R. Knutti, personal communication, 2008). The possibility that the variability is due to sampling statistics of the model runs can be examined from the results presented by FASM from an ensemble of 5 124-year runs with the GISS-ER model, for which a considerably narrower range of values was reported, with mean 23.9 W a m^{-2} K^{-1} and 26.8 W a m^{-2} K^{-1} for analysis by the bisector and ratio of slopes methods of S07, respectively, and corresponding ranges 21.4 - 25.7 and 24.3 - 30.2 W a m^{-2} K^{-1}. These results suggest that the large model-to-model differences in effective global heat capacity found by KKFA are not due to sampling issues but rather reflect true model-to-model differences. It is clear therefore that these models cannot all be providing an accurate representation of the processes that govern Earth's heat uptake in response to forcings. It would thus seem at the very least that comparison with observations should help to identify models which represent global heat uptake with greater accuracy and perhaps point the way to identifying the reasons for this. In the present context it might not be inappropriate to conclude that at least some of the differences between models and observations must be attributed to model inaccuracies.

Autocorrelation of GMST. Analogous to examination of the heat capacities, comparison of the autocorrelative properties of time series GMST from models with those drawn from observations, as was done by Wigley et al. [1998], would seem to provide further useful insights into the fidelity with which climate models can simulate Earth's climate system. Figure 1*a* of FASM, which compares the dependence of autocorrelation on lag time for the five ensemble members of the GISS-ER calculations with that from the GISS observational data set shows that the members of the model ensemble all exhibit autocorrelation that decreases considerably more rapidly with increasing lag time than is the case for the observations.

Determination of time constants. As found by Scafetta [NS08] and as discussed above, examination of the monthly observational data reveals a time constant of ~0.4 a in addition to the longer time constant of interest here, ~8.5 a, that must be accounted for in the determination of the longer time constant. Use of the monthly data also provides many more data points which, when plotted according to Figure 2, can reveal systematic departures from the two time-constant model. In recognition of this, it seems that a next useful step would be to examine the model monthly data to ascertain the extent to which the model data exhibit behavior similar to the observational data. A great advantage of model experiments is that the data from long (multi-century) control runs can be used for this examination to avoid issues associated with the short duration of the observational data set and which might reveal even longer time constants that are not revealed in the ~125 a observational record or in similarly short records of model data. FASM note that the GISS-ER model takes a number of decades to equilibrate after application of external forcing, and similar behavior is noted in many models that participated in the so-called "commitment" experiment (KKFA, Figure 1*b*). However other models exhibit rather shorter time constants. Brasseur and Roeckner [2005] using the Hamburg coupled atmosphere-ocean model found that GMST relaxed to a new equilibrium state following a step function perturbation in forcing with a time constant of about 12 a, and Matthews and Caldeira [2007], using an intermediate complexity global model with explicit representations of ocean circulation and heat uptake, found global surface temperature to relax following a step function perturbation with a time constant of about 5 a. While, as KKFA point out, the temperature excursion following an impulse forcing, such as shortwave forcing following a single volcanic eruption, can be accurately simulated by models having a large range of time constants, a climate system time constant that is constrained by autocorrelation over an extended time period may be useful in identifying models that exhibit time responses that are, or are not, characteristic of Earth's climate system.

Climate sensitivity. The key motivation for S07 was to determine climate sensitivity empirically, from observational data over the instrumental record without independent knowledge of the forcing, which is highly uncertain, mainly because of uncertainty in aerosol forcing [IPCC, 2007]. Again, while the approach is empirical, it would seem to be usefully informed by comparisons with model results. It would thus be instructive to ascertain the extent to which equation (1) relating climate sensitivity to climate system time constant and effective heat capacity holds in models for which all three quantities are well known.

Concluding remarks

The continuing high uncertainty associated with estimates of Earth's climate sensitivity pertinent to climate change on the multidecadal time scale has motivated an effort to determine this sensitivity empirically within an energy balance framework. The several Comments have raised important questions over the applicability of this method, especially in the context of the limited record of reliable estimates of global mean surface temperature and global ocean heat content and multiple time constants characterizing climate system response to perturbations and have led to an extension of the approach of S07 that can identify and deal with the consequences of short term (subannual) autocorrelation on the quantification of the effective climate system time constant. This further analysis has solidified the basis for the empirical determination of climate sensitivity and leads to upward revision of the estimated climate system time constant by about 70% over that given in S07, to 8.5 ± 2.5 a. This upward revision results in an increase in climate sensitivity λ_s^{-1} to 0.51 ± 0.26 K/(W m^{-2}), corresponding to an equilibrium temperature increase for doubled CO_2 $\Delta T_{2\times} = 1.9 \pm 1.0$ K.

Recently it was shown [Roe and Baker, 2007], as had been recognized earlier (e.g., Lindzen and Giannitsis, [1998]) that it is difficult to precisely determine climate sensitivity in climate models because slight changes in the climate system feedback factor resulting from changes in parameterizations of physical processes can result in large changes in modeled climate sensitivity, especially as the positive feedback approaches unity. This finding led to the observation [Allen and Frame, 2007] that climate sensitivity may not be a very useful quantity and the suggestion that the quest for determining this quantity be called off. The difficulty of determining climate sensitivity by climate models due to the strong dependence of modeled climate sensitivity to model parameters should not be taken as diminishing the utility of this quantity. Rather this difficulty of determining climate sensitivity by climate models should be viewed as a strong argument for empirical determination of this quantity from observables of Earth's climate system, as was the objective of S07.

The value of the climate system sensitivity determined by the empirical approach of S07, revised as presented here, is more consistent with the best estimate of this sensitivity presented by the recent assessment report of the IPCC [2007], $\Delta T_{2\times}$ = 3.0 (+1.5/-1) K (66% probability) than the value given by S07, λ_s^{-1} = 0.30 ± 0.14 K/(W m^{-2}), corresponding to $\Delta T_{2\times}$ = 1.1 ± 0.5 K. Attention is called also to other recent independent estimates of climate sensitivity that are likewise at the low end of the IPCC [2007] range: 0.29 to 0.48 ± 0.12 K/(W m^{-2}) [Chylek et al., 2007]; 0.49 ± 0.07 K/(W m^{-2}) [Chylek and Lohmann, 2008]; and 0.65 ± 0.28 K/(W m^{-2}) [Scafetta and West, 2007]; the latter investigators also suggested the climate system time constant pertinent to increase in Northern Hemisphere temperature is 9 ± 3.25 a, consistent with the present result.

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Table 1. Time constant, a, of climate system as inferred from autocorrelation of global mean surface temperature as tabulated ~~in~~ by the Goddard Institute of Space Studies [GISS, NASA, USA; Hansen *et al.*, 1996; updated at http://data.giss.nasa.gov/gistemp/; MS denotes Meteorological Station data set; LO denotes Land-Ocean data set;] and by the Climatic Research Unit [CRU, University of East Anglia, UK; Brohan et al., 2006; updated at http://cdiac.esd.ornl.gov/trends/temp/jonescru/jones.html] data sets (after deseasonalization by subtraction of monthly means), evaluated from the slope of the graph of the logarithm of the autocorrelation coefficient $r(\Delta t)$, where Δt is the lag time, versus lag time, for autocorrelation coefficient evaluated conventionally, or incorporating a correction for bias due to the short duration of the time series estimated by the method of Quenouille [1949].

	From slope		Visual inspection		Double exponential	
Data Set	Conventional	Unbiased	Conventional	Unbiased	Conventional	Unbiased
GISS MS 1880-2007 Global	8.6 ± 0.7	9.0 ± 0.4	9 ± 3	9 ± 2	8.6 ± 0.3	10.8 ± 0.3
GISS MS 1880-2007 NH	8.6 ± 0.8	9.1 ± 0.5	9 ± 3	9 ± 2	8.5 ± 0.4	10.8 ± 0.3
GISS MS 1880-2007 SH	5.1 ± 1.8	8.1 ± 1.8	9 ± 4	9 ± 4	4.6 ± 0.5	7.2 ± 0.5
GISS LO 1880-2007 Global	7.7 ± 0.4	7.9 ± 0.3	8.5 ± 2	8.5 ± 2	9.7 ± 0.5	10.1 ± 0.4
CRU 1880-10/2007 Global	7.1 ± 0.3	5.7 ± 0.1	7 ± 1	5.5 ± 0.5	8.2 ± 0.3	7.6 ± 0.2
CRU 1880-10/2007 NH	11.6 ± 1.4	9.3 ± 0.8	12 ± 3	9 ± 3	9.8 ± 0.6	9.7 ± 0.5
CRU 1880-10/2007 SH	4.8 ± 0.3	4.8 ± 0.1	4.5 ± 1	5 ± 1	5.9 ± 0.3	6.6 ± 0.2

Table 2. Empirical determination of key properties of Earth's climate system. Revision of Table 3 of S07 taking into account the increase in estimate of climate system time constant τ from 5 to 8.5 a and resultant increase in climate sensitivity λ_s^{-1} from 0.30 to 0.51 K/(W m^{-2}).

Quantity	Unit	Value	Uncertainty
Effective global heat capacity C	W a m^{-2} K^{-1}	16.7	7
Effective global heat capacity C	GJ m^{-2} K^{-1}	0.53	0.22
Effective climate system time constant τ	a	8.5	2.5
Equilibrium climate sensitivity λ_s^{-1}	K/(W m^{-2})	0.51	0.26
Equilibrium temperature increase for doubled CO_2 $\Delta T_{2\times}$	K	1.9	1.0
Increase in GMST over twentieth century $\Delta T_{s,20}$ [Folland *et al.*, 2001]	K	0.57	0.085
Total forcing over twentieth century F_{20}	W m^{-2}	1.1	0.6
Lag in temperature change over twentieth century ΔT_{lag}	K	0.05	
Total greenhouse gas forcing over twentieth century $F_{G,20}$ [IPCC, 2001, Figure 6.8]	W m^{-2}	2.2	0.3
Forcing in twentieth century other than greenhouse gas forcing ΔF_{20}	W m^{-2}	-1.1	0.7
Temperature increase in twentieth century due to greenhouse gas forcing	K	1.1	0.6
Temperature increase in twentieth century due to CO_2 forcing	K	0.6	0.3
Temperature decrease in twentieth century due to other than greenhouse gas forcing	K	-0.5	0.4
Total forcing by well mixed greenhouse gases 1750-1998 [IPCC, 2001]	W m^{-2}	2.43	0.24
Temperature increase 1750-1998 due to greenhouse gas forcing	K	1.2	0.6

1 **Figures**

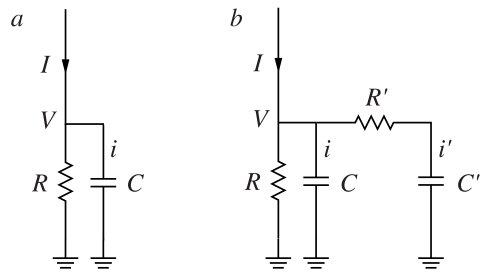

Figure 1. Equivalent electrical circuits for determination of climate sensitivity. *a*) single capacitance, single time constant; *b*) two capacitances and two time constants.

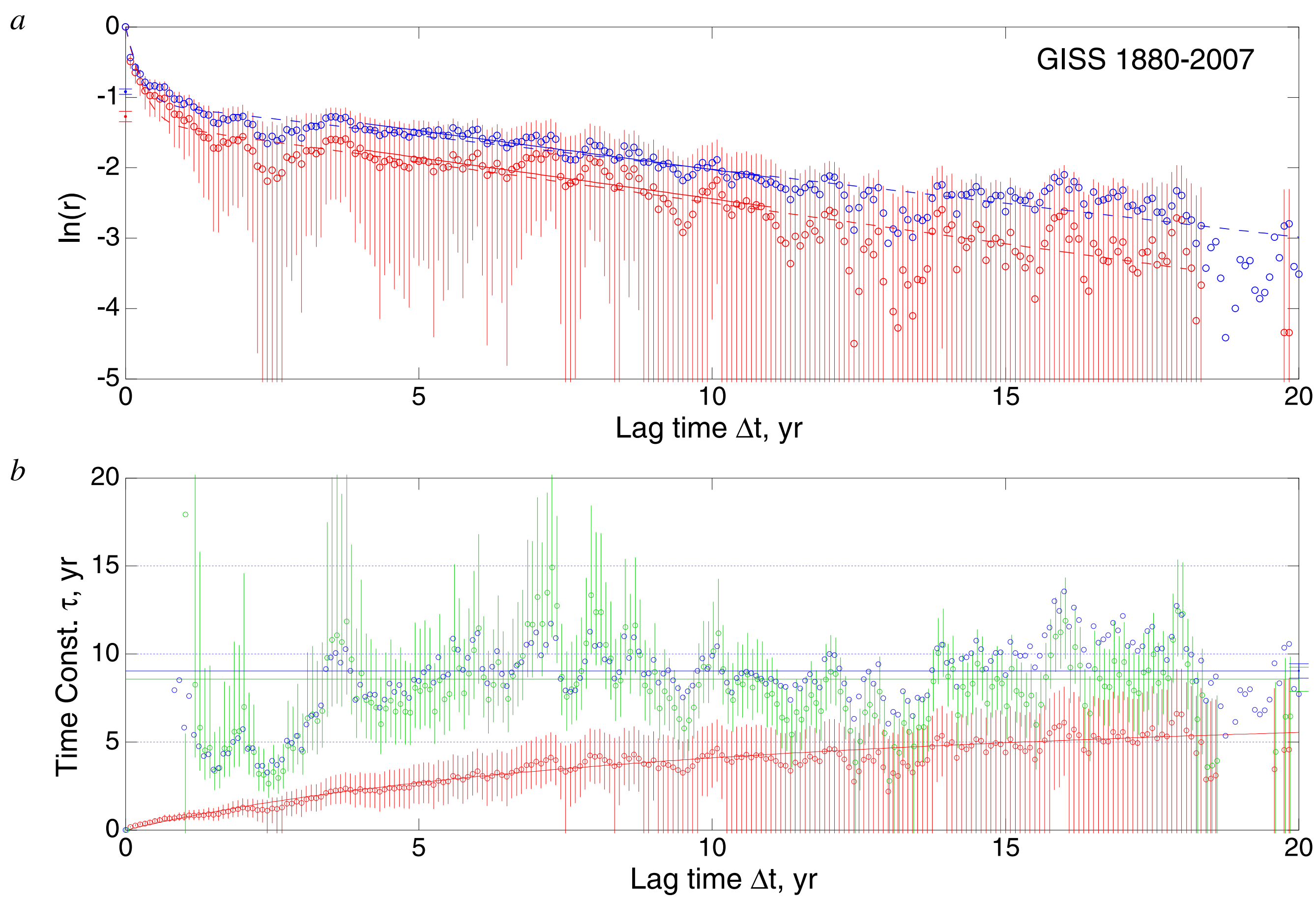

Figure 2. Dependence of autocorrelation *r* of monthly average global mean surface temperature on lag time Δt and corresponding time constant for the GISS Global Meteorological Station data set (1880-2007). *a*) Semi-logarithmic plot of *r* as evaluated conventionally (as in S07, red) and by the method of Quenouille [1949] (blue) to correct for bias due to finite duration of time series. Red and blue line segments denote linear regression fits to the data over the range (4-11 a) indicated by their extent; $\Delta t = 0$ intercepts and regression uncertainties are shown on left axis. Dashed curves show fit to a double exponential as proposed by Scafetta [NS08]. Uncertainties on *r* represent estimated standard deviation evaluated as the square root of the estimated variance of *r* evaluated according to Bartlett [1946]. *b*) Climate system time constant evaluated as $\tau(\Delta t) = -\Delta t / \ln r(\Delta t)$ for the raw autocorrelation coefficients and linear fit (red) as in S07, and for the autocorrelation coefficients corrected by the $\Delta t = 0$ intercepts of fits in *a* (green for raw autocorrelation data; blue, for bias-corrected data). Horizontal green and blue lines (and propagated uncertainties from regressions, right axis) indicate climate system time constant evaluated from slopes of fits in *a* as $\tau = -1/d\ln r(\Delta t)/d\Delta t$. Uncertainties on τ are propagated from uncertainties on *r*. Data are deseasonalized by subtracting the mean January from all January values, etc.